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Digital Signal Processing

The Elegance of Hamming Codes, and Where it Falls Apart

The elegance of the Hamming Code comes in two different ways. The first is the scaling of the amount of bits being checked and the second provides a shortcut for the identification of bit errors. Hamming Codes are quite elegant because they are able to scale to larger messages very easily because of the way the redundant bits are placed. The redundant bits are in the base 2 spots, so if there was a large data set of 2*N* bits, there would be *N* redundant bits located at positions 1, 2, 4, 8, 16, 32, and so on. For example a 28 or 256 bit array will have redundant bits in the 1, 2, 4, 5 16, 32, 64, and 128 positions. We can also see that the space between these bit positions gets larger as the data set becomes bigger, so the number of redundant bits required for the message scales logarithmically, which is very useful. This means that we do not need to use up much data storage for correcting errors, and as those messages get larger, the amount relative to the whole message gets smaller. The second elegancy of the Hamming codes is the location of the error using the ‘exclusive or’ (xor) method. If we take the xor of the location of each bit, we are essentially taking the parity of the array in one operation. The result of the xor operation will yield the binary number of the location of the error in the array. This is much easier to execute in a computer software than the parity checks, making Hamming Codes fast to complete. While elegant, there is a flaw in the Hamming Code. Unfortunately, it is built on the assumption that there will only be 1 error within the bit array. If there are 2 errors, then the parity checks will not work and the Hamming Code method falls apart. Similarly, the FFT video described a discrete Fourier Transform for data, so in order to accurately determine the frequencies of the signal, the sample rate must be high enough to collect 2 samples per wavelength. If this is not satisfied, then the amplitude of that frequency cannot be determined within the signal. The FFT operates on this assumption that the sample rate is high enough to provide a large enough window to capture all relevant frequencies.